LITERATURE CITED

1. B. A. Rozenberg and N. S. Enikolopyan, "Polymer composite materials," Zh. Vses. Khim. Obshch., 25, No. 5, 524-530 (1980).
2. I. I. Perepechko, Acoustical Methods for the Investigation of Polymers [in Russian], Khimiya, Moscow (1973).
3. B. I. Sazhin (ed.), Electrical Properties of Polymers [in Russian], Khimiya, Leningrad (1977).
4. W. P. Mason (ed.), Physical Acoustics, Vol. 1A: Methods and Devices, Academic Press, New York (1964).
5. A. Lid'yard, Ion Conductivity of Crystals [Russian translation], IL, Moscow (1962).
6. V. I. Alekseenko, V. I. Krylovich, P. N. Logvinovich, and A. I. Silenko, "Automated system for the detection and recording of frequency–phase shifts of continuous ultrasonic waves," in: Nondestructive Physical Testing Methods and Facilities (Abstracts), Sec. A: Acoustical Methods, Part 2 [in Russian], Minsk (1981), pp. 74-75.
7. I. I. Perepechko and L. A. Bodrova, "Molecular mobility in polytetrafluoroethylene," Vysokomol. Soedin. B, 9, No. 2, 116-118 (1967).
8. I. I. Perepechko and L. A. Bodrova, "Determination of the temperature transitions in polymer materials by an acoustical method," Plast. Massy, No. 7, 56-57 (1967).
9. I. I. Perepechko and L. A. Bodrova, "Investigation of temperature transitions in polymer materials at low frequencies by an acoustical method," in: Application of Ultrasonics to the Study of Matter [in Russian], No. 22, MOPI, Moscow (1967), pp. 62-66.
10. A. S. Freidin and L. B. Orlova, "Technique for determining the degree of hardening of polymers," Plast. Massy, No. 1, 72-73 (1963).
11. G. D. Andrievskaya, High-Test Oriented Glass-Reinforced Plastics [in Russian], Nauka, Moscow (1966), pp. 72-80.
12. A. A. Konkin, "Heat-resistant carbon fibers," in: High-Temperature Heat-Resistant and Incombustible Fibers [in Russian], Khimiya, Moscow (1978), pp. 217-340.

HEAT FLOW THROUGH FIBER-REINFORCED COMPOSITE LAYER

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UDC 536.248

The problem of heat flow through a layer with a finite number of rows of periodically positioned fibers all oriented along the same direction is solved.

1. Let a layer of thickness h contain s infinite rows of cylindrical fibers, whose longitudinal axes are parallel to one another and to the flat boundary of the layer. The radius of a fiber in the p-th row equals R_p and we shall denote the distance between two neighboring fibers in a row by a. Let us introduce a Cartesian system of coordinates so that the z axis coincides with the longitudinal axis of one of the fibers in the first row, while the y axis is perpendicular to the boundaries of the layer. The coordinates of the center of the k-th fiber in the p-th row in the system Oxy are $(-x_p^0 + ka, \quad -y_p^0)$, where $k = 0, \quad \pm 1, \quad \pm 2, \ldots, p = \overline{1, s}$; $(-x_p^0, -y_p^0)$ are the coordinates of the center of a fiber in the p-th row closest to the origin of coordinates. We shall also introduce the notation $z_p^0 = x_p^0 + iy_p^0$, $h_p = y_p^0 - y_{p-1}^0$, $p = \overline{1, s}$; h_1 is the distance from the center of the fiber in the first row to the upper boundary of the layer; h_{s+1} is the distance from the center of the fiber in the s-th row to the lower boundary (Fig. 1).

Let us examine the following boundary value heat-conduction problem:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) t = 0;$$

$$\frac{\partial T}{\partial y}\bigg|_{y=h_1-h} = -q; \left[\frac{\partial T}{\partial y} + \beta (T - T_0)\right]\bigg|_{y=h_1} = 0; \tag{1}$$

Institute of Superhard Materials, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 3, pp. 417-424, September, 1982. Original article submitted May 27, 1981.

$$T(R_p, \varphi_{pk}) = t_{pk}(R_p, \varphi_{pk});$$

$$\lambda_0 \frac{\partial T}{\partial \rho_{pk}}\Big|_{\rho_{pk}=R_p} = \lambda_p \frac{\partial t_{pk}}{\partial \rho_{pk}}\Big|_{\rho_{pk}=R_p}; \quad p = \overline{1, s}; \quad -\infty < k < \infty. \tag{1}$$

In relations (1), (ρ_{pk}, φ_{pk}) are the coordinates of a point in the polar coordinate system, fixed to the k-th fiber of the p-th row; in addition t = T(x, y) in the region $h_1 - h \leq y \leq h_1$ and $\rho_{pk} \geq R_p$, $t = t_{pk}(\rho_{pk}, \varphi_{pk})$ in the region $\rho_{pk} \leq R_p$. The coordinates x_{pk}, y_{pk}, ρ_{pk} are measured in units of R_1. In addition, it is assumed that $x_{p0} \equiv x_p$, $y_{p0} \equiv y_p$.

The first group of boundary conditions corresponds to fixing the heat flux $Q = \lambda_0 q/R_1$ at $y = h_1 - h$ and heat transfer to the surrounding medium (whose temperature is T_0) occurs according to the Newton–Rikhman law with heat-transfer coefficient $\alpha = \lambda_0\beta/R_1$ through the surface $y = h_1$. The second group of conditions corresponds to an ideal thermal contact between the layer and the fibers and, in addition λ_0 is the coefficient of thermal conductivity of the layer material, λ_p, $p = \overline{1, s}$ is the coefficient of thermal conductivity of the material of the fibers in the p-th row.

2. For constant q and T_0, the temperature field T(x, y) is a periodic function of x with period *a*. For this reason, we shall represent it in the form

$$T = A + By + 2\,\mathrm{Re}\,(T),$$

$$T_1 = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} A_n^{(k)}\xi_n(z + z_k^0) + \sum_{p=1}^{\infty} B_p \exp[-i\psi_p(z - ih_1)] + \sum_{p=1}^{\infty} C_p \exp[i\psi_p(z + i(h - h_1))], \tag{2}$$

where z = x + iy; $\psi_p = (2\pi/a)\,p$, while the system of solutions ξ_n of Laplace's equation has the form

$$\xi_n(\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{(\omega - ak)^n} = \begin{cases} \sum_{m=0}^{\infty} \xi_{nm} \exp(i\psi_m\omega), & \mathrm{Im}(\omega) > 0; \\ \sum_{m=0}^{\infty} \bar{\xi}_{nm} \exp(-i\psi_m\omega), & \mathrm{Im}(\omega) < 0; \end{cases}$$

$$\xi_{nm} = \frac{2\pi}{a}\,\varepsilon_m \psi_m^{n-1} \frac{(-i)^n}{(n-1)!}; \quad \varepsilon_0 = \frac{1}{2}; \quad \varepsilon_m = 1\ (m \geq 1). \tag{3}$$

We introduce the following notation:

$$a_m^{(k)} = \sum_{n=1}^{\infty} A_n^{(k)}\xi_{nm}, \quad b_m^{(k)} = \sum_{n=1}^{\infty} A_n^{(k)}\bar{\xi}_{nm}, \quad k = \overline{1, s}.$$

Satisfaction of the conditions on the flat layer boundaries leads to the following relations:

$$B = -q; \quad \beta A + B(1 + \beta h_1) = \beta\left[T_0 - 2\,\mathrm{Re}\left(\sum_{p=1}^{s} a_0^{(p)}\right)\right];$$

$$\left[\sum_{p=1}^{s} \bar{b}_m^{(p)} \exp(i\psi_m \bar{z}_p^0)\right] \exp[-\psi_m(h - h_1)] = C_m - \bar{B}_m \exp(-\psi_m h); \tag{4}$$

$$\left[\sum_{p=1}^{s} a_m^{(p)} \exp(i\psi_m z_p^0)\right](\psi_m - \beta)\exp(-\psi_m h_1) = \bar{B}_m(\psi_m + \beta) + C_m(\beta - \psi_m)\exp(-\psi_m h).$$

The temperature distribution in all fibers of the p-th row is the same, so that $t_{pk}(\rho_{pk}, \varphi_{pk}) = t_p(\rho_{pk}, \varphi_{pk}) = t_p(\rho_p, \varphi_p)$ and it is sufficient that the joining conditions be satisfied for the fiber with k = 0. We shall represent the temperature in this fiber by the series

$$t_p = \sum_{n=-\infty}^{\infty} D_n^{(p)} \rho_p^{|n|} \exp(in\varphi_p), \quad D_{-n}^{(p)} = \bar{D}_n^{(p)}. \tag{5}$$

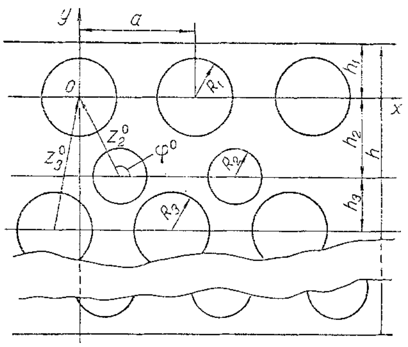

Fig. 1. Diagram showing the distribution of fibers in a layer (z = 0 cross section).

The solution for the layer in polar coordinates (ρ_p, φ_p) has the form

$$T = A + B\left[\rho_p\left(\frac{\exp(i\varphi_p) - \exp(-i\varphi_p)}{2i}\right) - \operatorname{Im}(z_p^0)\right] + \tag{6}$$
$$+ 2\operatorname{Re}\left\{\sum_{n=1}^{\infty} A_n^{(p)}\frac{\exp(-in\varphi_p)}{\rho_p^n} + \sum_{n=0}^{\infty}\left(\beta_n^{(p)} + \gamma_n^{(p)} + \sum_{q=1}^{s}\alpha_n^{(p)(q)}\right)\rho_p^n\exp(in\varphi_p)\right\},$$

where

$$\alpha_n^{(p)(q)} = \sum_{t=1}^{\infty} A_t^{(q)}\alpha_{nt}^{(p)(q)};$$

$$\alpha_{nt}^{(p)(q)} = \frac{(-1)^n}{n!}\,\frac{(n+t-1)!}{(t-1)!}\,\xi_{n+t}(z_p^0 - z_q^0);\quad p \neq q;$$

$$\alpha_{nt}^{(p)(p)} = [(-1)^n + (-1)^t]\,\frac{(n+t-1)!}{n!\,(t-1)!}\,\frac{1}{a^{n+t}}\sum_{k=1}^{\infty}\frac{1}{k^{n+t}};$$

$$\beta_n^{(p)} = \frac{(-i)^n}{n!}\sum_{m=1}^{\infty}\psi_m^n\exp(-\psi_m h_1)\,B_m\exp(i\psi_m z_p^0);$$

$$\gamma_n^{(p)} = \frac{i^n}{n!}\sum_{m=1}^{\infty}\psi_m^n\exp[-\psi_m(h-h_1)]\,C_m\exp(-i\psi_m z_p^0).$$

From the joining conditions for the temperature fields in the layer and a fiber in the p-th row, we arrive at the equalities:

$$D_0^{(p)} = A + B(-\operatorname{Im} z_p^0) + 2\operatorname{Re}\left(\beta_0^{(p)} + \gamma_0^{(p)} + \sum_{q=1}^{s}\alpha_0^{(p)(q)}\right);$$
$$\frac{i}{2}B\delta_n^1 R_p + \frac{A_n^{(p)}}{R_p^n} + \left(\bar\beta_n^{(p)} + \bar\gamma_n^{(p)} + \sum_{q=1}^{s}\bar\alpha_n^{(p)(q)}\right)R_p^n = D_{-n}^{(p)}R_p^n; \tag{7}$$
$$\frac{i}{2}B\delta_n^1 R_p - n\frac{A_n^{(p)}}{R_p^n} + n\left(\bar\beta_n^{(p)} + \bar\gamma_n^{(p)} + \sum_{q=1}^{s}\bar\alpha_n^{(p)(q)}\right)R_p^n = \frac{\lambda_p}{\lambda_0}nD_{-n}^{(p)}R_p^n;\quad n = \overline{1, \infty};\quad p = \overline{1, s}.$$

Relations (4) and (7) together form a closed infinite system of linear algebraic equations. If the unknowns B_n, C_n, and $D_n^{(p)}$ are eliminated from the system, then we obtain a system with the unknowns $A_n^{(p)}$:

$$\frac{i}{2}B\delta_n^1 R_p\left(\frac{\lambda_p}{\lambda_0} - 1\right) + \left(\frac{\lambda_p}{\lambda_0} + 1\right)\frac{A_n^{(p)}}{R_p^n} + \tag{8}$$
$$+ R_p^n\left(\frac{\lambda_p}{\lambda_0} - 1\right)\sum_{q=1}^{\infty}\sum_{t=1}^{\infty}\{A_t^{(q)}[\eta_{nt}^{(p)(q)} + \varkappa_{nt}^{(p)(q)}] + \bar A_t^{(q)}[\alpha_{nt}^{(p)(q)} + \beta_{nt}^{(p)(q)} + \gamma_{nt}^{(p)(q)}]\} = 0;\quad n = \overline{1, \infty};\quad p = \overline{1, s},$$

where

$$\beta_{nt}^{(p)(q)} = \sum_{m=1}^{\infty}\frac{i^n}{n!}\psi_m^n(\psi_m - \beta)\frac{\exp(-2\psi_m h)}{\Delta_m}\exp[i\psi_m(z_q^0 - z_p^0)]\,\xi_{tm};$$

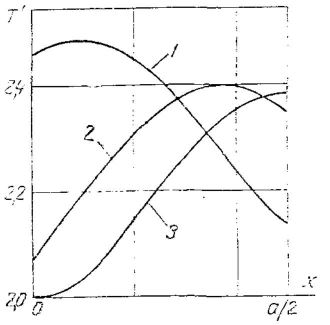

Fig. 2

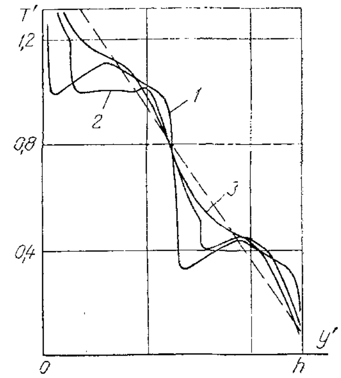

Fig. 3

Fig. 2. Temperature distribution at the lower surface of the layer (T′, x are dimensionless quantities): 1) $\varphi^\circ = 0.35\pi$; 2) 0.45π; 3) 0.5π.

Fig. 3. Temperature distribution over the thickness of a layer containing two rows of fibers (T′, y′ are dimensionless quantities): 1) x = 0; 2) 0.4a; 3) 0.5a.

$$\gamma_{nt}^{(p)(q)} = \sum_{m=1}^{\infty} \frac{(-i)^n}{n!} \psi_m^n (\psi_m - \beta) \frac{\exp(-2\psi_m h)}{\Delta_m} \exp[i\psi_m(\bar{z}_p^0 - \bar{z}_q^0)]\, \bar{\xi}_{tm};$$

$$\eta_{nt}^{(p)(q)} = \sum_{m=1}^{\infty} \frac{i^n}{n!} \psi_m^n (\psi_m - \beta) \frac{\exp(-2\psi_m h_1)}{\Delta_m} \exp[i\psi_m(z_q^0 - \bar{z}_p^0)]\, \xi_{tm};$$

$$\varkappa_{nt}^{(p)(q)} = \sum_{m=1}^{\infty} \frac{(-i)^n}{n!} \psi_m^n \frac{(\psi_m + \beta)}{\Delta_m} \exp[-2\psi_m(h - h_1)] \times$$

$$\times \exp[i\psi_m(\bar{z}_p^0 - z_q^0)]\, \bar{\xi}_{tm};$$

$$\Delta_m = \psi_m + \beta + (\beta - \psi_m)\exp(-2\psi_m h).$$

As is evident from the above presentation, the infinite system of algebraic equations (8) after replacing the unknowns $y_n^{(p)} = \left(\frac{\lambda_p}{\lambda_0} + 1\right)\frac{A_n^{(p)}}{R_p^n}$ transforms into a normal-type system with the conditions that the surfaces of the fibers and the flat boundaries of the layer do not touch. This property makes it possible to solve the system approximately by the method of reduction [1].

3. As an example, we shall examine the problem of heat flow through a layer with thickness $h = h_1 + h_2 + h_3$, containing two rows of fibers with radii R_1 and R_2 (Fig. 1). The calculations were performed with the following values of the thermophysical parameters: $\lambda_1 = \lambda_2 = 10\lambda_0$, $\beta = 10$. We examined two variants of the geometric parameters of the problem: 1) $a = 3R_1$, $a = 2.4R_1$, $R_2 = R_1$, $h_1 = h_3 = 1.2R_1$, $h_2 = 2.4R_1$, $\varphi^0 = 0.35\pi$, 0.45π, 0.5π; 2) $\pi(R_1^2 + R_2^2) = 0.63ah$, $\varphi^0 = 0.3\pi$; a) $R_2 = R_1$, $h_1 = h_3 = 1.1R_1$, $h_2 = 1.9R_1$; b) $R_2 = 2R_1$, $h_1 = 1.1R_1$, $h_3 = 2.1R_1$, $h_2 = 2.7R_1$; c) $R_2 = 0.5R_1$, $h_1 = 1.05R_1$, $h_2 = 1.35R_1$, $h_3 = 0.55R_1$. In order to determine the temperatures in the layer and the fibers, we used expressions (2) and (5), in which the highest value of the index was equal to 10. This corresponded to retaining in the system (8) 20 complex equations, which ensured high accuracy of the solution of the problem (the error in satisfying the joining conditions (1) is fractions of a percent).

The results of the calculations are shown in Figs. 2-4, where graphs of the dimensionless temperature distribution $T' = (T - T_0)/T_0$ are presented. Figure 2 corresponds to the first variant of the geometric parameters with $a = 3R_1$. Here, the temperature distribution is shown at points on the lower boundary of the layer, through which the external flux Q enters, as a function of the displacement angle of the rows of fiber. From these graphs follows the result that the maximum value of the temperature depends on the angle φ^0. In addition, the lowest temperature level occurs for the layer with $\varphi^0 = 0.5\pi$ (the fibers in the rows are above one another), while the highest level occurs when the fibers in the rows are shifted by the half-period $a/2$. The curves in Fig. 3 correspond to a temperature distribution over the layer thickness in its

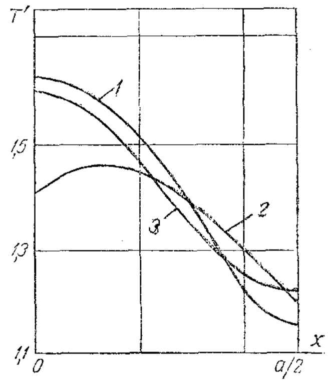

Fig. 4. Temperature distribution at the lower layer boundary with constant concentration: 1) $R_2/R_1 = 0.5$; 2) 1.0; 3) 2.0.

transverse sections x = 0, x = 0.4a, and x = 0.5a with $a = 2.4R_1$ and $\varphi^0 = 0.5\pi$ (the remaining parameters are the same as in Fig. 2). Here, $y' = y + h_2 + h_3$. The data in the graphs indicate the very nonmonotonic change in temperature over the thickness of the layer (compared to a uniform layer). The curve x = 0 has characteristic local maxima at points corresponding to the centers of the fiber cross sections.

The values of the temperature at points on the lower boundary of the layer are shown in Fig. 4 for two variants of the geometric parameters of the problem as a function of the ratio of the fiber radii in the first and second rows with constant fiber concentration in the layer equal to 0.63. It is characteristic that the minimum temperature level is observed in the case when the fibers in both rows have the same radius. From Figs. 4 and 2 we arrive at the conclusion that the average temperature at the layer boundary, through which external heat flow is introduced, depends considerably on the geometrical parameters of the composite layer examined. This result is of definite practical significance in problems of heat removal from composite materials.

4. If we use the rigorous solution of the problem of heat flow through a fibrous layer presented above, then we can examine the problem of transforming the given inhomogeneous layer to some fictitious, homogeneous, thermophysically equivalent layer. In order to obtain an effective coefficient of heat conduction, it is sufficient to establish, based on a rigorous solution, a relation between the average (within period a) values of the component of the heat flux q_y parallel to the Oy axis and the derivative $(\partial/\partial y)T$.

Let us represent the average values $(\partial/\partial y)T$ and q_y in the form

$$\left\langle \frac{\partial}{\partial y} T \right\rangle ah = \int\limits_{F_M} \frac{\partial T}{\partial y} dxdy + \int\limits_{F_1} \frac{\partial t_1}{\partial y} dxdy + \int\limits_{F_2} \frac{\partial t_2}{\partial y} dxdy;$$

$$- \langle q_y \rangle ah = \lambda_0 \int\limits_{F_M} \frac{\partial T}{\partial y} dxdy + \lambda_1 \int\limits_{F_1} \frac{\partial t_1}{\partial y} dxdy + \lambda_2 \int\limits_{F_2} \frac{\partial t_2}{\partial y} dxdy; \tag{9}$$

$$F_M + F_1 + F_2 = ah, \quad F_1 = \pi R_1^2, \quad F_2 = \pi R_2^2 .$$

Applying Green's equation, using (2) for the temperature in the layer and (5) for the temperature in the fiber, as well as the condition for the temperatures to be equal at the boundary between the layer and fiber, after some transformations, we obtain:

$$\left\langle \frac{\partial}{\partial y} T \right\rangle ah = Bah - 2\pi i \left(A_1^{(1)} - \bar{A}_1^{(1)} + A_1^{(2)} - \bar{A}_1^{(2)}\right);$$

$$- \langle q_y \rangle ah = \lambda_0 Bah. \tag{10}$$

Defining the effective coefficient of thermal conductivity as the ratio of $-\langle q_y \rangle$ and $\langle (\partial/\partial y)T \rangle$, we have from (10)

$$\lambda_{\text{eff}} = \lambda_0 \frac{1}{1 - \dfrac{4\pi}{ah} \operatorname{Im}\left(A_1^{(1)} + A_1^{(2)}\right)} . \tag{11}$$

In (11), $A_1^{(1)}$, $A_1^{(2)}$ are the first unknowns of the infinite system (8) with q = 1.

Table 1 presents the values of $\lambda_{\text{eff}}/\lambda_0$ for the two variants of the geometrical parameters of the layer described above. It follows from the table that the value of the effective coefficient of thermal conductivity of the layer is determined both

TABLE 1. Effective Coefficients of Thermal Conductivity for a Layer with Two Rows of Fibers

c	0,43				0,63		
φ^0	0,3	0,35	0,45	0,5	0,3	0,3	0,3
R_2/R_1	1,0	1,0	1,0	1,0	2,0	1,0	0,5
λ_{eff}/λ_0	2,179	2,181	2,265	2,284	2,524	3,390	3,079

by the concentration of fibers in the layer and their thermal conductivity and by the geometrical parameters of the layer. Figure 3 shows the temperature distribution over the thickness of a fictitious homogeneous layer with $\lambda_0 = \lambda_{eff}$ (dashed line). As can be seen from the figure, the solution of the problem for such a layer gives a satisfactory approximation for the average values of the temperature at the layer boundaries and can be used in approximate calculations. At the same time, we note that the difference between the approximate and exact solutions at interior points of the layer (especially in fibers) is very large.

It is interesting to compare the effective coefficients of thermal conductivity of the fibrous layer presented in the table and the analogous coefficients for a composite medium reinforced with fibers [2, 3]. The effective coefficients of thermal conductivity calculated from the results in [2] for a composite medium with tetragonal and hexagonal packing of fibers are $\lambda_{eff}/\lambda_0=2{,}12\left(c=0.43,\quad \varphi^0=\frac{\pi}{2}\right)$, $\lambda_{eff}/\lambda_0=2.10\left(c=0.43,\ \varphi^0=\frac{\pi}{3}\right)$, and $\lambda_{eff}/\lambda_0=3.18\left(c=0.63,\ \varphi^0=\frac{\pi}{3}\right)$. The given values are close to those presented in Table 1 with $R_2/R_1 = 1$, $\varphi^0 = \pi/2$, and $\varphi^0 = 0.3\,\pi$. The reduction problem was solved in [3] for a composite material with random positioning of unidirectional fibers. Since the results of this paper are presented in graphic form, quantitative comparison is difficult. We shall only indicate the fact that the values of λ_{eff}/λ_0 with c = 0.43 show a greater difference (the disagreement is about 10%) than for c = 0.63 (the disagreement is about 4%).

In conclusion, we note the following. The three-dimensional heat-conduction problem for a thin plate, reinforced with cylindrical rods, was reduced in the first approximation to a two-dimensional problem in [4] using an asymptotic integration method. The corresponding heat-conduction equation contains the parameter B $(\lambda_1, \lambda_2, \Gamma_1)$ which is defined in terms of the solution of the auxiliary problem of the type examined above. For the latter problem, the flat layer boundaries are assumed to be thermally insulated, while the conditions for contact between the layer and the fibers correspond to equality of the temperatures and a given jump in the heat fluxes on the contact surface. Thus, the method presented in this paper can be used to establish finally the two-dimensional equation of heat conduction in a thin plate with finite dimensions in a plane reinforced by one or several rows of cylindrical fibers.

NOTATION

h, thickness of the layer; a, distance between neighboring fibers in a row; R_i, radius of a fiber in the i-th row; x_i, y_i, Cartesian coordinates; ρ_i, φ_i, polar coordinates in a system of coordinates fixed to the i-th row of the fiber; T, temperature in the layer; t_i, temperature in a fiber on the i-th row; T_0, temperature of the surrounding medium; α, coefficient of heat transfer; c, fiber concentration in the layer; φ^0, relative displacement angle of rows of fibers; Q, heat flux; q_y, component of the heat flux for the dimensionless problem, parallel to the Oy axis; λ_0, coefficient of thermal conductivity of the layer material; λ_i, coefficient of thermal conductivity of a fiber in the i-th row; λ_{eff}, effective coefficient of thermal conductivity of the fibrous layer.

LITERATURE CITED

1. L. V. Kantorovich and V. I. Krylov, Approximate Methods of Higher Analysis [in Russian], Fizmatgiz, Moscow (1962).
2. N. I. Nikityuk, "A method for solving problems of mechanics of fibrous materials," Proceedings of the 2nd Republican Conference of Young Scientists on Mechanics, Naukova Dumka, Kiev (1979), pp. 157-161.
3. Yu. A. Buevich and V. G. Markov, "Stationary transport in fibrous composite materials," Inzh.-Fiz. Zh., 36, No. 5, 828-834 (1979).
4. E. Kh. Lokharu and E. A. Tropp, "Heat conduction by a reinforced plate," Inzh.-Fiz. Zh., 35, No. 3, 497-505 (1978).